Department of Mathematics
Mathematics Math3062
Non-linear dynamics and chaos
BD/2/2005

ASSIGNMENT 8

For assessment: due on or before Tuesday October 18th

Be sure to give adequate explanation and data for all answers.

Question 1 requires analytic work only, questions 2 & 3 are numerical.

1. This question involves examples of Cantor sets produced by iteration from $I_0 = [0, 1]$. In lectures I considered the example where the open middle 1/3 is iteratively removed from each component sub-interval.
(i) Consider the fractal constructed by removing the open middle 1/2 from each component sub-interval. For a covering, choose intervals of length $\epsilon_k = 1/4^k$. What are the numbers of intervals of length ϵ_k required to cover I_0, I_1 and I_2, for $k = 0, 1, \cdots$?
(ii) What are the numbers N_k of intervals of length ϵ_k required to cover I_∞ for general $k \geq 0$? Give an argument to support your answer.
(iii) Using the result of part (ii), show that the capacity dimension is $d_C = 1/2$.
(iv) Use a similarity argument to find an equation relating $N(\epsilon)$ and $N(4\epsilon)$, and use it to recover the capacity dimension without employing any explicit counting.

2. Examine orbits of the Henon map at $a = 1.07$ and $b = 0.3$ using the `Iterations (2d)` option in *Chaos for Java*.
(i) There are in fact *two* coexisting attractors. Find each by using initial point $(0, 0.25)$ for the one and $(0.6, -0.25)$ for the other. [Open two windows, one for each, so you can view them side by side.]
(ii) Remembering that the choice of initial point is crucial to finding the desired attractor, compute the Lyapunov exponents for $1.0 \leq a \leq 1.1$ using each of the two initial points for the iterations.
(iii) Compute also the Lyapunov dimension for each when $a = 1.07$.
(iv) Compute also the Fourier spectra for each when $a = 1.07$.
(v) What conclusions can you draw about the nature of each of the attractors? Explain your reasons and support them with appropriate observations made in parts (ii–iv).

[Hint: to get reasonably accurate numerical results, discard the first 10000 iterations when computing Lyapunov exponents and Fourier spectra, also use 1000 samples for the former and at least 10000 samples for the spectra.]

3. This assignment is about the Lozi map with (default) parameter values $a = 1.7$, $b = 0.3$,

(i) Verify that it has a strange attractor. (Use evidence from Fourier spectra and Lyapunov exponents.)

(ii) Estimate the Lyapunov dimension of the attractor.

(iii) Using box counting, with boxes of size 0.00025, 0.0005, ... 0.008, and orbits of length up to 10^7, estimate the capacity dimension of the attractor, consturcting a table of counts similar to the one presented in lectures for the Hénon map.