Department of Mathematics
Mathematics Math3062
Non-linear dynamics and chaos
BD/2/2005

ASSIGNMENT 1

Tutorial session August 1st — not for assessment

1. Consider the "Cubic #2" map

$$f(x) = rx(1 - x^2)/\sqrt{3}, \qquad (0 \le r \le 4.5)$$

(i) Show that f is a unimodal map of the interval $[0, 1]$ to itself for the given range of the parameter r.
(ii) Solve the fixed point equation, and show that there is only one fixed point ($x_0^* = 0$) for $0 \le r < \sqrt{3}$, but that there are two fixed points when $\sqrt{3} < r \le 4.5$. Give a formula for the new point x_1^*.
(iii) Derive a formula for $f'(x_1^*)$ as a function of r, and show that x_1^* is stable for $\sqrt{3} < r < 2\sqrt{3}$ and unstable for $2\sqrt{3} < r \le 4.5$.

2. It is always a good idea to check any formulae against some numerical calculations, if possible.
Using the "Graphical analysis" option, with the same "Cubic #2" map as in the previous question, find all fixed points of f, and the derivative value for each, for values of r between 1.0 and 4.0, in steps of 0.5. Tabulate your data, and use it as a cross-check on the formulae you derived.

3. In 1976, Otto Rössler suggested investigating the equations

$$\frac{dx}{dt} = -y - z, \qquad \frac{dy}{dt} = x + \alpha y, \qquad \frac{dz}{dt} = \alpha + z(x - \mu),$$

which have the simple property that their solutions circle the vertical (z) axis, lying close to the x-y plane much of the time. Where is the non-linearity?
Rössler chose $\alpha = 1/5$, and this is the default choice in *Chaos for Java*. Use the "Return Maps" window with $\mu = 5.7$ to view the maximum in x return map for these equations. Compare with the results shown in lectures for the Lorenz equations.