

Australian National University  
Canberra Australia

Special Year on Algebraic Geometry and Topology

## **Conference on Topology**

14 – 25 July 2003

# Abstracts

## Conference on Topology Abstracts

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**Speaker:** Mathew Ando

**Title:** Elliptic curves and algebraic topology

**Abstract:** The term "elliptic cohomology" refers to a family of generalized cohomology theories which are associated in a natural way with elliptic curves. These cohomology theories moderate a fascinating relationship between modular forms and theta functions, stable homotopy theory, and string theory. They are the receptacles for "elliptic genera".

I will begin by discussing why elliptic cohomology is an appealing object of study from the point of view of homotopy theory. Then I will describe how elliptic genera arise in string theory, and explain some of the predictions which arise from this point of view. Finally I will show how elliptic cohomology provides a natural setting for mathematical proofs of these predictions.

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**Speaker:** Jon Berrick

**Title:** Recent progress on the Bass trace conjecture

**Abstract:** The talk gives a historical survey on the conjecture, leading up to research with Chatterji & Mislin that ties in with V Lafforgue's work on the Bost conjecture.

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**Speaker:** Stephen Bigelow

**Title:** The Lawrence-Krammer representation of braid groups

**Abstract:** The Lawrence-Krammer representation is a faithful representation of the braid groups into a group of matrices. I will outline an elementary combinatorial proof of this. Then I will go into the connections with configuration spaces, homology, "twisted" Poincare duality, and the Iwahori-Hecke algebra.

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**Speaker:** David Blanc

**Title:** Rectifying homotopy commutative diagrams of spaces

**Abstract:** A problem which arises in many contexts in homotopy theory is the following: can we replace a diagram of topological spaces and maps which commutes only up to homotopy by a strict diagram of topological spaces? This comes up, for instance, when we try to determine when an H-space is actually (homotopy equivalent to) a topological group.

We shall give a survey on two basic approaches to the problem-one geometric, in terms of higher homotopy operations, and the other algebraic, in terms of cohomology groups-and try to explain how they are related.

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**Speaker:** Bea Bleile

**Title:** Homotopy Classification and Realization of  $PD^3$ -Pairs

**Abstract:** Generalizing Turaev's proof of Hendriks' Classification Theorem for  $PD^3$ -complexes we provide invariants which classify  $PD^3$ -pairs up to oriented homotopy equivalence. We discuss how Turaev's Realization Theorem for  $PD^3$ -complexes may be generalized to  $PD^3$ -pairs.

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**Speaker:** Simone Borghesi

**Title:** homotopy theory and degree formulae

**Abstract:** This talk will indicate how we can use motivic stable homotopy theory to define certain characteristic numbers for singular algebraic varieties satisfying an assumption. We will also sketch the proof of the necessary degree formulae for the Rost-Voevodsky program to the Bloch-Kato conjecture in algebraic K-theory from the existence of these characteristic numbers.

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**Speaker:** Danny Calegari

**Series title:** Foliations and dynamics in low dimensions

**General abstract for the three talk series:** Monotonicity is an important principle in topology; it is generally associated with 1- and 2- dimensional phenomena. In the theory of low dimensional manifolds, it manifests itself typically through the study of codimension 2- and 1- objects, for example in the context of foliations and dynamics, and also in the context of asymptotic geometry. The theme of this lecture series is that a surprising range of topological phenomena can be reduced to, and understood in terms of, groups of homeomorphisms of the circle.

**First talk title:** Foliations and asymptotic geometry

**Abstract:** In the first lecture, we introduce the universal circle associated to a taut foliation of an atoroidal 3-manifold. This circle collates and tames the circle at infinity of every leaf in a coherent way. By studying the action of the fundamental group on this circle, we show that the foliation admits a transverse essential lamination, that the fundamental group is word-hyperbolic, and that the manifold is homotopy rigid; i.e. every self-automorphism homotopic to the identity is isotopic to the identity.

**Second talk title:** Laminations and groups of homeomorphisms of the circle

**Abstract:** In the second lecture, which represents joint work with Nathan Dunfield, we give a dual construction which associates to a tight lamination with solid torus guts an action of the fundamental group on a circle. This construction makes use of a technical tool - the filling lemma - which says that a lamination with solid torus guts can be filled in to a lamination with solid torus complementary regions. This has applications to non-existence results for tight laminations and pseudo-Anosov flows.

**Third talk title:** Planar groups and circular groups

**Abstract:** In the third lecture, we study general group actions on the plane, and suggest a general correspondence between planar groups and circular groups. We show

under many circumstances that a group of homeomorphisms of the plane is abstractly isomorphic to a group of homeomorphisms of the circle. There are connections with Zimmer's conjecture, the Milnor-Wood inequality, Cannon-Thurston theory, and generalized braid groups.

**Speaker:** Wojciech Chacholski

**Title:**  $v$ -torsion spaces

**Abstract:** A topological space is said to be  $v$ -torsion if all of its  $v_n$ -periodic homotopy groups vanish, and its ordinary homotopy groups are, in a certain sense, concentrated at some chosen prime  $p$ . In this talk I will discuss certain global properties of the collection of  $v$ -torsion spaces. I will also present several motivating examples.

**Speaker:** James Coffey

**Title:** 3-manifolds built from three handlebodies

**Abstract:** Classically, two handlebodies glued together form a Heegaard splitting of a 3-manifold  $M$ . This is roughly equivalent to the structure coming from a Morse function on  $M$ . We study when three handlebodies are glued to form a 3-manifold  $M$ , so that each handlebody has fundamental group which injects into the fundamental group of  $M$ . This can be viewed as a generalisation of the construction of Seifert fibred spaces. Such 3-manifolds have many nice properties and appear to be very common.

**Speaker:** Diarmuid Crowley

**Title:** Quadratic modules and surgery theory

**Abstract:** I shall show how to extend the machinery of Matthias Kreck's modified surgery theory using the Baues' notion of a quadratic module. This gives rise to new classification results for certain classes of manifolds as well as a general framework in which to restate many existing results on the classification of manifolds.

**Speaker:** Alexei Davydov

**Title:** Rational homotopy types and Hochschild cohomology

**Abstract:** It is well known that deformation (or extension) theory of  $A_\infty$ -algebras is described by Hochschild cohomology. For example, sufficient conditions for formality of  $A_\infty$ -structures can be expressed in terms of vanishing of certain Hochschild cohomology. In another example the Lie algebra of the (pro-unipotent part of the) group of  $A_\infty$ -automorphisms appear as a subalgebra of the Hochschild cohomology.

It was proven by Kadeishvili that (commutative variants of)  $A_\infty$ -algebras model rational homotopy types. We will use some simple examples to illustrate how Hochschild (or rather Harrison) cohomology can be useful in checking formality or describing maps between rational homotopy types.

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**Speaker:** Yakov Eliashberg

**Title:** Three Lectures on Applications of Holomorphic Curves

**Abstract:** The lectures will be devoted to applications the theory of holomorphic curves, and more specifically of Symplectic Field Theory (SFT) to different problems in symplectic and contact topology, as well as to the theory of integrable systems. In particular, there will be discussed restrictions on the topology of Lagrangian embeddings, construction of invariants of contact manifolds. There will also be discussed an algebraic formalism of SFT and appearance of commuting differential operators in this context.

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**Speaker:** Ian Grojnowski

**Title:** *TBA*

**Abstract:** *TBA*

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**Speaker:** Joost van Hamel

**Title:** Towards intersection homology for real algebraic varieties.

**Abstract:** Intersection homology is an important topological invariant of singular complex varieties. The definition does not extend to arbitrary real algebraic varieties. In this talk I will describe my attempt to use techniques from equivariant cohomology to extract a good intersection homology (with 2-torsion coefficients) for real algebraic varieties from the intersection homology of their complexifications. I will also briefly discuss a completely different approach proposed by McCrory and Parusiński.

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**Speaker:** Lars Hesselholt

**First talk title:** The cyclotomic trace

**Abstract:** This talk will introduce the cyclotomic trace and survey recent results including the proof of the Lichtenbaum-Quillen conjecture for complete discrete valuation fields.

**Second talk title:** Laurent polynomial extensions

**Abstract:** I will give an explicit formula for the trace invariant of a Laurent polynomial ring in terms of that of the coefficient ring. This leads to a description in terms of the trace invariant of the topological Whitehead space of the circle.

**Third talk title:** Motivic complexes

**Abstract:** In this final talk I will discuss the analogy between the known results on the trace invariants and results/conjectures in algebraic K-theory. This suggests that there exist an analog for the trace invariant of motivic cohomology. In the end I will present a candidate for this new theory.

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**Speaker:** Jonathan Hillman

**Title:** Minimal models for certain  $PD_4$ -complexes

**Abstract:** Let  $X$  be a finite  $PD_4$ -complex with group  $G$ . We show that if  $G$  is free or is one-ended and has a finite 2-dimensional  $K(G, 1)$  complex then there is a 2-connected degree 1 map from  $X$  to a “minimal” such complex. If  $G$  is a surface group the minimal complex is the total space of an  $S^2$ -bundle over  $K(G, 1)$ , and is Determined by  $X$ .

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**Speaker:** Kenshi Ishiguro

**Title:** Homotopy fixed point sets and actions on homogeneous spaces of  $p$ -compact groups

**Abstract:** We generalize a result of Dror Farjoun and Zabrodsky on the relationship between fixed point sets and homotopy fixed point sets, which is related to the generalized Sullivan Conjecture. As an application, we discuss extension problems considering actions on homogeneous spaces of  $p$ -compact groups.

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**Speaker:** Bernhard Keller

**Title:** Derived invariance of higher structures on the Hochschild complex

**Abstract:** The Hochschild complex of an associative algebra carries actions of various operads, the richest one being the brace operad. We show that an equivalence between the derived categories of the module categories of two algebras yields an isomorphism between their Hochschild complexes viewed as objects of the homotopy category of algebras over the brace operad.

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**Speaker:** Henning Krause

**Title:** The telescope conjecture for derived categories

**Abstract:** The telescope conjecture for derived categories is the formal analogue of a conjecture from stable homotopy theory which is due to Bousfield and Ravenel. The failure of this conjecture is the motivation to study cohomological quotients of triangulated categories, a concept which generalizes Verdier’s classical construction of a quotient modulo a triangulated subcategory. In my talk I shall discuss specific examples of such quotients, and I will show their relevance for non-commutative localizations. It turns out that the telescope conjecture is relevant for the problem of lifting chain complexes up to homotopy along a ring homomorphism. This lifting problem is motivated by some applications in algebraic K-theory; it has been studied in recent work of Neeman and Ranicki, and I shall present some progress.

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**Speaker:** Andrew Kricker

**Title:** Non-commutative localisation and the Kontsevich integral of a boundary link

**Abstract:** The Kontsevich integral is one of the mathematical models for the perturbative expansion of Witten's Chern-Simon path integral. Almost ten years of close scrutiny left us with a rich and textured theory which, unfortunately, had almost nothing to do with the topology of knots and three-manifolds. This is rather surprising for an invariant which purports to project a "complete" picture of knot theory. The good news is that the formal theory (which consists mostly of the drawing and gluing together, ad nauseam, of certain trivalent graphs) has progressed to a point where we are beginning to understand how the Kontsevich integral represents some geometric topology.

The result we will focus on is the following. There is a certain expansion of the Kontsevich integral of a boundary link (the "loop expansion") which collects certain infinite pieces of the Kontsevich integral into finite combinations of trivalent graphs each of whose edges is labelled by a power series in non-commuting variables, one variable per component. The result (math.GT/0105028 (in progress), generalising math.GT/0005284), which is a finiteness statement, is that these power series are actually "rational functions", which is to say, power series generated by "machines" (in the sense of Farber). These functions promise a means for establishing deeper connections with topology (e.g. in math.GT/0107220 they are applied to determining certain quantum invariants of covering spaces). The topological study of the loop expansion of the Kontsevich integral was initiated by the author in math.GT/9901029.

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**Speaker:** Gus Lehrer

**Title:** The topology of the variety of regular semisimple elements of a Lie group.

**Abstract:** The regular semisimple elements form an open dense subvariety of a Lie group (or algebra), but many interesting things happen on the nilpotent cone, e.g. the "Springer action" of the Weyl group on cohomology. These are accessible through interpolation via the "Grothendieck-Springer resolution". In this talk I'll describe some results (joint work with G.B. Segal) on the stable homology of the regular semisimple varieties, which use some unpublished work of his on equivariant homotopy, and describe some applications to reductive groups over finite fields (time permitting).

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**Speaker:** Mark Mahowald

**Title:** Some applications of the spectrum  $TMF$  in homotopy groups of spheres and cobordism.

**Abstract:** The refined Hopkins-Miller spectrum  $TMF$  was expected to open several avenues toward understanding  $v_2$  periodic homotopy of spheres and other spectra. In this note, I will discuss some of the recent progress in this direction and some of the spectra which are related. I will also discuss some recent applications.

The talk should be of interest to a general topological audience. I hope to be able to keep the technical details to a minimum.

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**Speaker:** Jack Morava

**Title:** Two-categories of four-manifolds

**Abstract:** Four-manifolds, smooth and otherwise, are an extremely rich subject, whose study has grown enormously in the last two decades. This talk is an attempt to organize some of those developments along lines suggested by work of Segal, Tillmann, and others on conformal field theory as the representation theory of a two-category of two-manifolds.

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**Speaker:** Fernando Muro

**Title:** On the proper homotopy classification of locally compact  $A_n^2$ -polyhedra

**Abstract:** The algebraic invariants used by J. H. C. Whitehead to classify (stable) homotopy types of simply connected 4-dimensional polyhedra are the cellular chain complex and the (Steenrod) Pontrjagin invariant. Proper homotopy theory is a middle step between the homotopy and homeomorphism classification of spaces. Algebraic tools in proper homotopy theory have been developed by several authors (Baues-Quintero, Beattie, Edwards-Hastings, Farrell-Wagoner), such as the proper cellular chain complex. However the fact that the “proper” algebra has projective dimension 2 motivates important differences with ordinary homotopy theory. In this talk we shall show that in general there are not Pontrjagin-Steenrod invariants in proper homotopy theory. For this we construct new proper cohomological invariants and compute one of them in a purely algebraic way from the proper cellular chain complex by using quadratic algebra. We shall also give a stable proper homotopy classification theorem for properly simply connected 4-dimensional locally compact polyhedra with less than 3 ends, and exhibit a proper Moore space of degree 2 which is not a proper co-H-space.

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**Speaker:** Frank Neumann

**Title:** Etale homotopy of moduli stacks of algebraic curves with symmetries

**Abstract:** Using the machinery of etale homotopy theory a la Artin-Mazur we determine the etale homotopy types of certain moduli stacks over  $\bar{\mathbb{Q}}$  parametrizing algebraic curves of genus  $g > 1$ , having fixed finite subgroups of its automorphism groups, which in the associated complex analytic situation are given as finite subgroups of the mapping class group of genus  $g > 1$ . This is a variation of a theme by Oda. This is joint work with P. Frediani (Pavia).

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**Speaker:** Walter Neumann

**Title:** The Casson Invariant Conjecture

**Abstract:** This is joint work with J. Wahl. The Casson Invariant Conjecture (CIC) asserts that for a complete intersection surface singularity whose link is an integral homology sphere, the Casson invariant of that link is one-eighth the signature of the Milnor fiber. We study a class of such complete intersections called “splice type.” This class

conjecturally includes all Gorenstein singularities with homology sphere link, up to equisingular deformation. We propose, and verify in a nontrivial case, a stronger conjecture than the CIC for splice type singularities: a precise topological description of the Milnor fiber. We prove the CIC itself for a special class of splice type singularities which includes all previously proven cases of the Conjecture.

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**Speaker:** Paul Norbury

**Title:** Closed geodesics on Riemmanian spheres

**Abstract:** This is joint work with Hyam Rubinstein. We construct closed geodesics on spheres that contain points where the curvature blows up, extending the result for smooth spheres. We do this via a minimax approach—in a family of loops on the sphere, take the maximum length loop, then minimise this maximum length loop with respect to all families of loops. An application of this result is the existence of minimal hypersurfaces in toric manifolds.

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**Speaker:** Andrew Percy

**Title:** Eckmann-Hilton duality of primary homotopy and cohomology operations

**Abstract:** The talk will look at the duality of  $n$ -ary homotopy and cohomology operations together with the groups they operate on. We will give examples of homotopy  $\Pi$ -algebras, which are the algebraic object consisting of the homotopy groups of a space and the primary operations on these groups. We then propose the Eckmann-Hilton dual, cohomology  $H$ -algebras, which are the algebraic objects consisting of the cohomology groups of a space with primary operations on them and give some further examples.

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**Speaker:** Andreas Rosenschon

**Title:** The Quillen-Lichtenbaum conjecture at the prime 2

**Abstract:** We give a proof of the conjecture. This is joint work with Paul-Arne Østvar.

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**Speaker:** Sayed Roushon

**Title:** Vanishing of Whitehead group of a class of group

**Abstract:** In this talk we show that the fibered isomorphism conjecture of Farrell and Jones corresponding to the stable topological pseudoisotopy functor is true for the fundamental groups of a class of complex manifolds and for a class of discrete groups. A consequence of this result is that the Whitehead group, reduced projective class groups and the negative  $K$ -groups of any of these groups vanish whenever the group is torsion free. We also prove the same results for a class of real manifolds.

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**Speaker:** Hyam Rubinstein

**Title:** Cubical models for 3-manifolds

**Abstract:** ‘Most’ 3-manifolds should have metrics of non-positive curvature, according to geometrisation. We present some results and ideas about finding finite cubical complexes with non-positive curvature homotopy equivalent to many 3-manifolds. This is joint work with I. Aitchison, M. Sageev and W. Lickorish

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**Speaker:** Mina Teicher

**Title:** *TBA*

**Abstract:** *TBA*

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**Speaker:** Ulrike Tillmann

**Title:** Topology of strings and the stable cohomology of mapping class groups.

**Abstract:** Twenty years ago, Mumford initiated the systematic study of the cohomology ring of moduli spaces of Riemann surfaces. Around the same time, Harer proved that the homology of the mapping class groups of oriented surfaces is independent of the genus in low degrees, increasing with the genus. The (co)homology of mapping class groups thus stabilizes. At least rationally, the mapping class groups have the same (co)homology as the corresponding moduli spaces. This prompted Mumford to conjecture that the stable rational cohomology of moduli spaces is generated by certain tautological classes that he defines. Much of the recent interest in this subject is motivated by mathematical physics and, in particular, by string theory. The study of the category of strings led to the discovery of an infinite loop space, the cohomology of which is the stable cohomology of the mapping class groups. This homotopy theoretic approach has recently led to the proof of the Mumford conjecture by Madsen and Weiss. An analysis of the category of strings in a background space leads to the formulation of a parametrized version of Mumford’s conjecture.

In the three lectures I hope to explain the background and the basic idea’s of the proof of Mumford’s conjecture. I will also discuss torsion information in the stable cohomology of the mapping class group, draw the connection with Gromov-Witten invariants, as well as put the theorem by Madsen and Weiss into a wider context.

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**Speaker:** Stephan Tillmann

**Title:** Boundary slopes and the logarithmic limit set

**Abstract:** The A-polynomial of a 1-cusped 3-manifold is generalised to an eigenvalue variety of a multi-cusped 3-manifold, and the set of strongly detected boundary slopes is determined in terms of Bergman’s logarithmic limit set.

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