The Cosmic Ladder.

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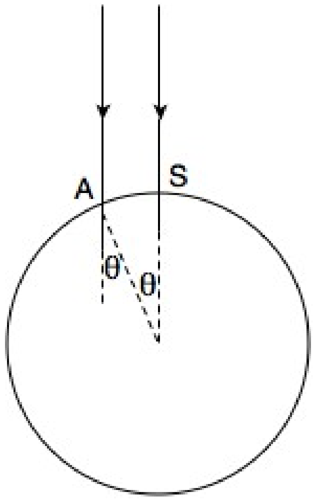
1. Radius of the Earth. Eratosthenes, 279–194 BC.
2. Distance to, and radius of, the Moon. Aristarchus, 310–230 BC.
3. Distance to, and radius of, the Sun. Aristarchus, 310–230 BC.
4. Distance to Mars, and other planets. Copernicus, 1473–1543.
5. Speed of light. Romer, 1644–1710; Huygens, 1629–1695.
6. Distances to nearby stars. Bessel, 1784–1846.
7. Distances to moderately distant stars. Hertzsprung, 1873–1967; and Russell, 1877–1957.
8. Distances to very distant stars. Leavitt, 1868–1921.
9. Shape of the Universe.

Radius of Earth, r_E. Eratosthenes, 279–194 BC.



Light from Sun. S = Syene, A = Alexandria. Distance from A to S is 740km; $\theta = 7^\circ$.

Let r_E be the radius of Earth

$$2\pi r_E \leftrightarrow 360^\circ, \quad 740 \leftrightarrow 7^\circ.$$

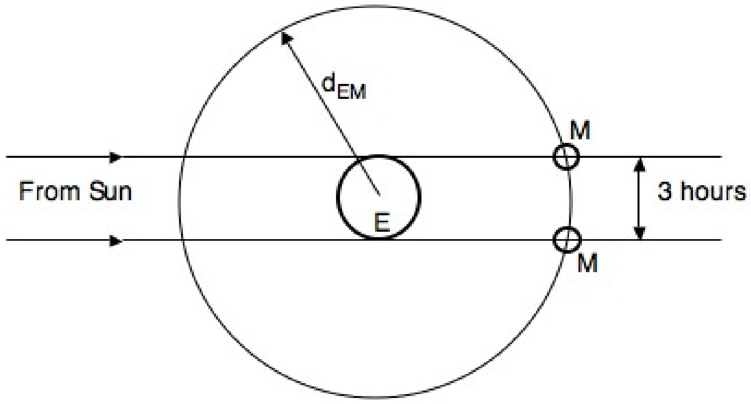
So

$$\frac{2\pi r_E}{740} = \frac{360}{7}$$

$$\therefore r_E = \frac{360 \times 740}{7 \times 2\pi} = 6067 \text{ km}.$$

(In fact, $r_E = 6376 - 6378$ km.)

Distance to Moon, d_{EM}. Aristarchus, 310–230 BC.



Eclipse of Moon. E = Earth, M = Moon.
M moves $2r_E$ km in 3 hours.

M moves:

$$2\pi d_{EM} \text{ km in 28 days,}$$
$$2r_E \text{ km in 3 hrs.}$$

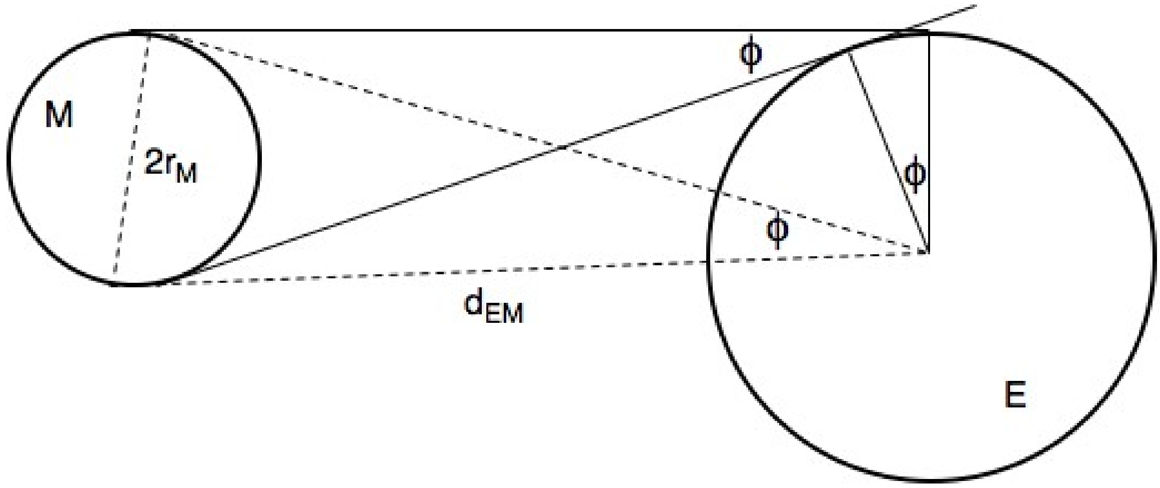
So:

$$\frac{2\pi d_{EM}}{2r_E} = \frac{28 \times 24}{3}$$

$$\therefore \quad d_{EM} = \frac{28 \times 24}{3\pi} r_E = \frac{28 \times 24}{3\pi} \times 6057 \approx 430\,000 \text{ km.}$$

(In fact, d_{EM} varies from 356 410 - 406 740 km.)

Radius of Moon, r_M. Aristarchus, 310–230 BC.



Setting Moon: *Earth rotates ϕ° in 2 minutes. Angles ϕ equal or close.*

Earth rotates : ϕ° in 2 minutes & 360° in 24 hours.

$$\text{So} \quad \frac{\phi}{360} = \frac{2}{24 \times 60}, \quad \therefore \phi = \frac{2 \times 360}{24 \times 60} \approx \frac{1}{2}^\circ.$$

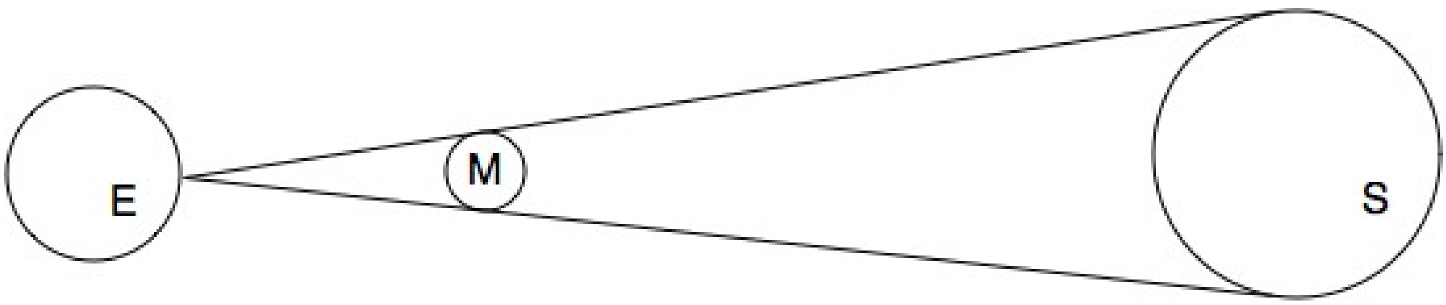
Sector of radius d_{EM} & angle ϕ subtends arc $2r_M$.

$$\therefore r_M = \frac{1}{2} d_{EM}\, \phi(\text{radians}) \approx 430\,000 \times \frac{1}{2} \cdot \frac{2\pi}{360} \approx 1876 \text{ km}.$$

(In fact, $r_M = 1738$ km.)

Distance d_{ES} to, radius r_S of, the Sun.
Aristarchus, 310–230 BC.

Note: One *astronomical unit*, 1AU, is defined to be d_{ES}.



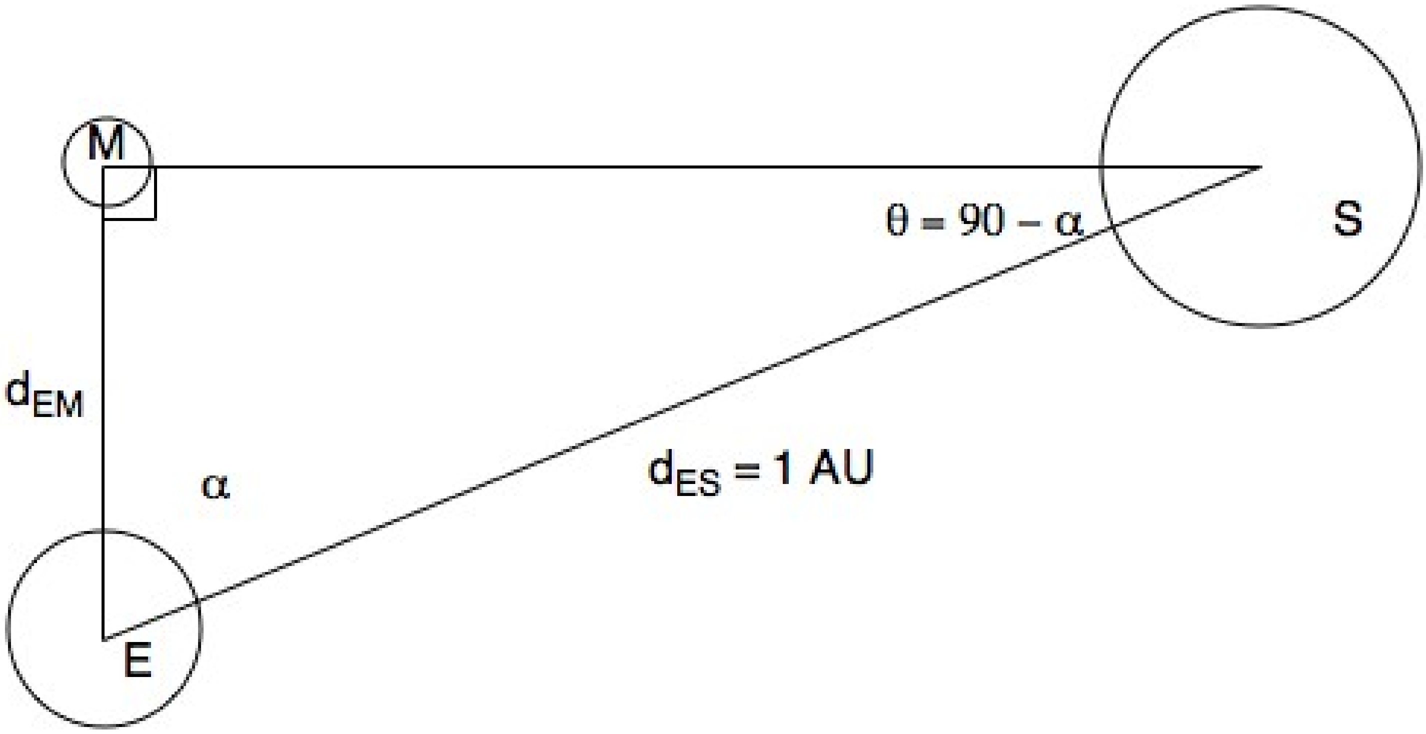
Eclipse of Sun: *M and S subtend roughly same angle on Earth.*

$$\therefore \frac{r_S}{d_{ES}} \approx \frac{r_M}{d_{EM}} \approx \frac{1876}{430\,000} \approx \cdot 0043.$$

(True value is $\cdot 0046 \ldots$.)

So once we can find one of r_S and d_{ES}, we can find the other.

Finding d_{ES}. Aristarchus, 310–230 BC.



Half moon during daytime: *Measure α.*
Aristarchus: $\theta \approx 3°$; true value: $\theta \approx \frac{1}{6}^{\circ}$.

$$1AU = d_{ES} = \frac{d_{EM}}{\sin\theta}$$

$$d_{ES} = \begin{cases} \approx \frac{430\,000}{\cdot 052} \approx 8.30 \times 10^6 \text{ km}, & \theta = 3^{\circ} \\ \approx \frac{430\,000}{\cdot 0029} \approx 1.48 \times 10^8 \text{ km}, & \theta = \frac{1}{6}^{\circ} \end{cases}$$

(True value is $1AU = d_{ES} \approx 1.50 \times 10^8$ km.)

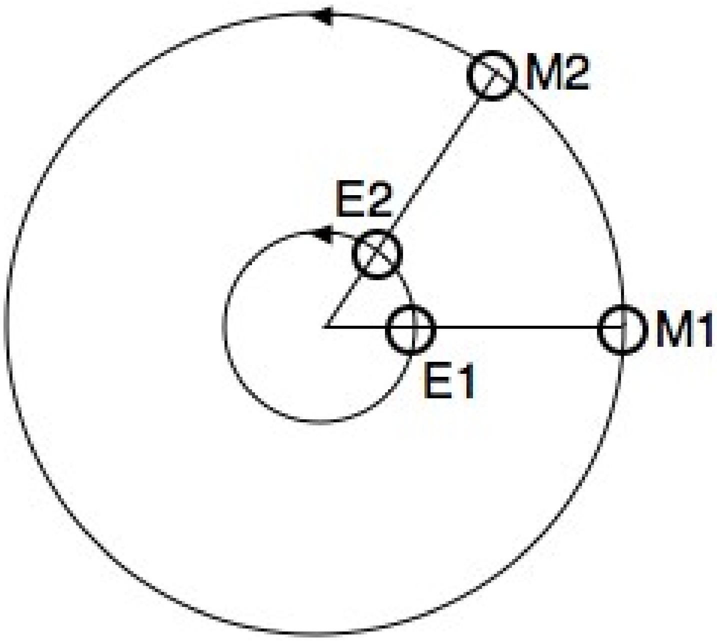
Finding r_S. Aristarchus, 310–230 BC.

From before:

$$r_S \approx \cdot 0043 \times d_{ES}$$
$$\approx \begin{cases} 3.5 \times 10^4 \text{ km}, & \theta = 3^\circ \\ 6.4 \times 10^5 \text{ km}, & \theta = \frac{1}{6}^\circ. \end{cases}$$

(True value is $6.955 \cdots \times 10^5$ km.)

Distance Sun to Mars. Copernicus, 1473–1543.



First find the angular velocity of Mars!

E has a greater angular velocity than Mars.

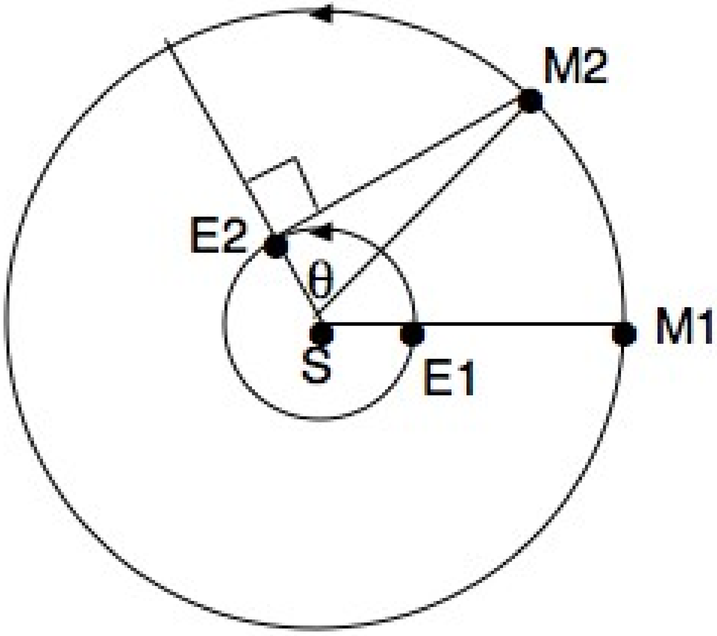
E "chases" Mars, and *first* "catches" Mars in same relative position after 780 (earth) days.

In 780 days, E moves $\frac{780}{365} \times 360° \approx 769°$, a bit over 2 revolutions.

It follows Mars moves a bit over 1 revolution (*why?*), namely $\approx 769° - 360° = 409°$.

So *Mars moves* $\frac{409}{780} \approx \cdot 524°$ *per (earth) day.*

Finding distance from Sun to Mars.



Initially E and Mars essentially line up with the Sun. After 106 days, Mars and Sun are at a right angle.

$$E \text{ moves } 106 \times \frac{360^\circ}{365} = 104.5^\circ$$

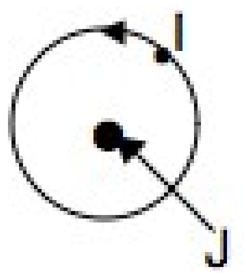
$$\text{Mars moves } 106 \times \cdot 524^\circ = 55.5^\circ$$

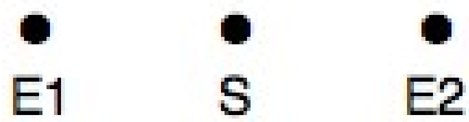
$$\therefore\ \theta = 104.5^\circ - 55.5^\circ = 49.0^\circ$$

$$\therefore\ d_{\text{S,Mars}} = \frac{d_{ES}}{\cos\theta} \approx 1.52\, d_{ES} = 1.52 AU.$$

(Correct to 2 decimals. Problem: good estimate 1 AU.)

Finding speed of light. (Used to find later distances.) Romer, 1644–1710; Huygens, 1629–1695.





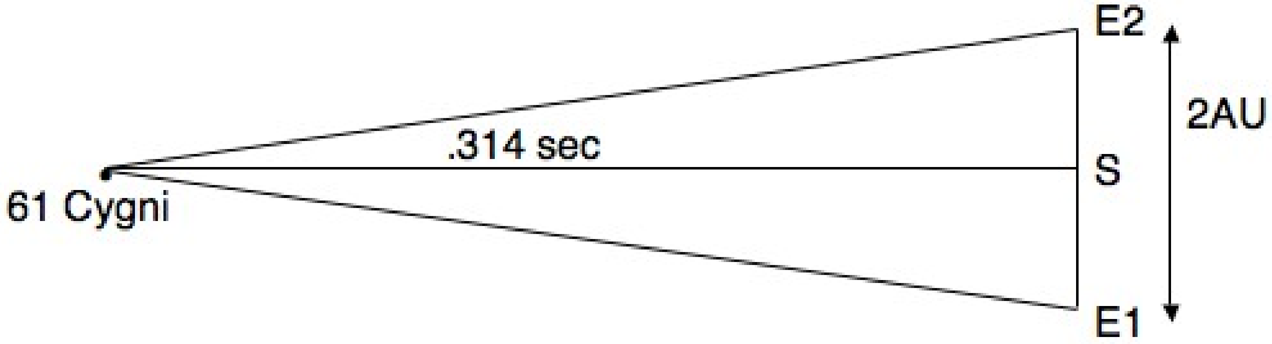
Io (I) rotates around Jupiter (J).
Cycle begins 20 minutes later if Earth (E) is at E2 than it "should" if E were at E1.

Conclusion: light takes 20 minutes to travel 2 AU.

$$\begin{aligned}\therefore \text{speed of light} &\approx \frac{2AU}{20 \text{ minutes}}\\ &\approx \frac{2 \times 1.50 \times 10^8}{20 \times 60}\,\text{km/sec.}\\ &= 250\,000 \text{ km/sec.}\end{aligned}$$

(Correct value is 299 792 km/sec.)

Distances to nearby Stars. Bessel, 1784-1846.



using observations at opposite sides of the sun, Bessel obtained angle subtended is $2 \times .314$ *sec.*

There are 3,600 seconds to a degree.

A second is roughly the angle subtended by the ball in a ballpoint pen, on the other side of a football field!

From diagram, distance to 61 Cygni is approx

$$\frac{1AU}{\sin(.314 \text{ secs})} \approx 9.8 \times 10^{13} \text{ km} \approx 10.4 \text{ light years}$$
$$\approx \frac{1}{.314} \text{ parsec} \approx 3.18 \text{ parsec.}$$

(Actual value is 11.2 light years, or 3.42 parsec.)

A light year is distance travelled by light in a year, a parsec is the distance of a star subtending an angle of 1 second in previous diagram.