MATH2406: MATHEMATICAL METHODS I (HONOURS)
– PROPOSED SYLLABUS –

Partial Differential Equations (21 lectures)

- Introduction: What is a PDE?, first order PDE's in 2 variables, second order PDEs (heuristic discussion), initial and boundary problems, well-posed problems, types of 2nd order PDE's (**3 lectures**)
- Waves and Diffusions: wave equation, causality, conservation of energy; diffusion equation, maximum principle, uniqueness of solution, energy method, stability, diffusion on the whole line (Green's function) (**3 lectures**)
- Reflections and Sources: diffusion on the half-line, reflections of waves, diffusion with a source, operator approach, wave equation with a source, characteristic coordinates, Stokes' theorem, operator method (**2 lectures**)
- Boundary Problems: wave equation, separation of variables, Dirichlet boundary conditions, Neumann boundary conditions; diffusion equation, mixed boundary conditions (**1 lecture**)
- Fourier series: sine series, cosine series, full Fourier series, complex form, orthogonaility and general Fourier series, complex eigenvalues, completeness, notions of convergence (L^2-convergence, pointwise convergence uniform convergence) (**3 lectures**)
- Harmonic functions: Laplace equation, maximum principle, uniqueness of the Dirichlet problem, invariance, 2D Laplacian in polar coordinates, 3D Laplace equation in polar coordinates, Poisson formula, mean value theorem, strong maximum principle, differentiability of solution, Laplace equation on exterior of a disk, Laplace equation on annulus, Laplace equation on wedge (**3 lectures**)
- Green's identities and Green's functions: Green's first identity, volume of ball/sphere in n dimensions, mean value theorem, strong maximum principle, uniqueness of Dirichlet problem, Dirichlet principle, Green's second identity, representation formula, Green's function, solution to the Poisson problem, Dirichlet problem on a half-space, Dirichlet problem on a ball (**3 lectures**)
- Wave equation in n dimensions: energy functional, causality, wave equation in 3D, wave equation in 2D (**2 lectures**)
- Diffusion equation in n dimensions (**0.5 lecture**)
- Eigenvalue equation for the Laplacian, minimum principle (**0.5 lecture**)

Distributions, Transforms, Complex Analysis and applications (15 lectures)

- Distributions: definition, convergence of distributions, derivative, generalization to n dimensions, Green's functions revisited: Laplace equation (**1 lecture**)
- Fourier transform: definition, properties, generalization to n dimensions, application to PDE's, Green's functions revisited (diffusion equation, wave equation, Laplace equation) (**3 lectures**)

- Complex Analysis: contour integration, analytic continuation, residue theory, integration around branch points (**6 lectures**)
- Laplace transforms: properties, Watson's lemma, the inversion integral, inversions involving residues and branch cuts, asymptotics, application to ODE's and PDE's (**3 lectures**)
- Integral equations: classification, degenerate kernels, difference kernels (**2 lectures**)