MATH 2405

Syllabus:

First Order Differential Equations (8 Lectures) Ch 1 - 3

Classification of DEs. Linear differential operators.
General solution of LDE with driving term.
Structure of solution.
Soution for Delta Function impulse. The one sided Green's Function. Examples from circuit theory.
Properties of Non-Linear DEs.
Existence and uniqueness of solutions
Solution methods. Exact equations and integrating factors.
Autonomous Equations. Critical points.
Stability of solutions.
Examples from population dynamics.
Introduction to Bifurcations. Saddle-node and Pitch-fork bifurcations.
First order planar dynamical systems and orbits of solutions.
Compartmental models and applications from physics and biology

Higher Order Differential Equations (8 Lectures) – Ch 4 and 5

Initial value problems.
Reduction of an nth order LDE to a system of first order LDEs.
Existence and uniqueness of solutions. Singular points.
Linear dependence and independence. The Wronskian.
Self-adjoint form of 2^{nd} order LDE. Abel's theorem.
General solution of homogeneous LDE.

Dimensionality of the solution space.
Particular integrals, and solution of non-homogeneous LDE.
Solution by factorising linear operators, and by repeated quadrature.
Variation of parameters, and reduction of order.
Green's function for 2^{nd} order Initial Value problem.
Forced damped oscillations. Resonance.
Applications to mechanical systems and circuits.
Power series solutions of ODE's.
Solution about Ordinary points, and regular singular points.
Airy's, Legendre's and Besell's equations.
Bessell functions of the first and second kind of any order.
The Gamma function.

Systems of Differential Equations (6 Lectures) – Ch 7 and 9
Systems of first order differential equations.
Autonomous systems. Critical points and stability
Predator-Prey problems.
Linearisation near critical points.
Systems of first order linear differential equations and solution methods.
Stability of solutions of linearised DEs.
Systems of second order linear differential equations.
Normal modes of oscillation with examples.
Coupled non-linear systems.
Stability of non linear systems.
Non-linear pendulum.
Hopf bifurcation.

Laplace Transforms (6 lectures) – Ch 6
Definition.
Functions of exponential order and convergence of Laplace transform.

Linearity of transform and inverse transform.
Tranforms and inverse transforms of standard functions.
Solutions to simple initial value problems using Laplace transforms.
Transform of step function and solving differential equations with discountinuous forcing function.
Composite IVPs. Solution smoothness.
Transform of unit impulse function
The convolution integral
Solution in terms of transfer function.
Relation to Green's function.

Boundary Value Problems (6 lectures) – Ch 11
Two point boundary value problems
Solution of 2^{nd} order homogeneous equation with homogeneous boundary conditions
Development of solution of inhomogeneous equation using Green's functions. Existence and uniqueness of solutions.
Sturm-Liouville boundary values problems and properties of eigenvalues and eigenfunctions.
Development of solutions of non-homogeneous DE in terms of orthogonal eigenfunctions.
Singular Sturm-Liouville problems.