THE AUSTRALIAN NATIONAL UNIVERSITY

MATH2306: Mathematical Methods I
Advanced Vector Calculus

Lecture Notes 2006

Peter Bouwknegt

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