

1. Analytic functions

The basic thrust here is to characterise the ‘analyticity’ of a collection of functions in algebraic and/or topological means. More generally, when is an abstract ring or algebra isomorphic to a ring or algebra of analytic functions over some manifold.

Requirements: Complex variables (possibly several), ring theory, functional analysis, possibly differential geometry.

References

- [1] I. Richards, Axioms for analytic functions, *Adv. Math.* **5** (1970), 311–338.

2. Operators which preserve the spectrum

It is a classical result that a linear surjection $\Phi : M_n(\mathbb{C}) \rightarrow M_n(\mathbb{C})$ which preserves the set of eigenvalues must be of the form $\Phi(x) = axa^{-1}$ or $\Phi(x) = a^t x a^{-1}$ for some invertible a . The project is to investigate what happens more generally in the context of continuous linear operators and Banach algebras.

Requirements: Functional analysis, complex variable. A knowledge of French would be an advantage, but is not mandatory.

References

- [1] B. Aupetit, Spectrum-preserving linear mappings between Banach algebras and Jordan-Banach algebras, *J. London Math. Soc.* **62** (2000), 917–924.

3. Spectral continuity

The map $\Phi : \mathcal{B}(H) \rightarrow \mathcal{P}(\mathbb{C}) : T \mapsto \sigma(T)$ which takes a bounded operator to its spectrum, is upper semicontinuous, but its continuity points form a proper dense G_δ -set.

Two problems thus arise:

1. at which points of $\mathcal{B}(H)$ is Φ in fact continuous?
2. what ‘continuity-type’ properties hold regarding Φ ?

Requirements: Functional analysis, subharmonic functions and possibly several complex variables for (2.)

References

- [1] B. Aupetit, *A primer on spectral theory*, Springer-Verlag, New York, 1991.
- [2] J.B. Conway & B.B. Morrel, Operators that are points of spectral continuity, *Integral Equations and Operator Theory* **2** (1979), 174–198.

4. Rich and narrow

Narrow operators are ones which are in a certain technical sense “small”, rich subspaces are ones which give a narrow quotient map. For example, compact operators are narrow. The properties of narrow operators are closely tied to the geometry of the underlying spaces, in particular properties of measures with values in the range space.

Requirements: functional analysis and measure theory.

References

- [1] A.M. Plichko and M.M. Popov, Symmetric function spaces on atomless probability spaces, *Dissertationes Math.* **306** (1992), 1-85.
Note that this is not held in the ANU library, however, I have a copy available.
- [2] V.M. Kadets and M.M. Popov, On the Liapounov convexity theorem with applications to sign-embeddings, *Ukrainian Math. J.* **44** (1992), 1091-1098.
- [3] V.M. Kadets, R.M. Shvidkoy and D. Werner, Narrow operators and rich subspaces of Banach spaces with the Daugavet property, *Studia Math.* **147** (2001), 269-298.