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1. Finite p -groups with many subgroups 2-subnormal

Groups in which every subgroup is normal were investigated by Dedekind as early as 1897. He showed that such groups are either abelian or the direct product of a quaternion group with a periodic abelian group with no elements of order 4. The property of being a normal subgroup is not transitive and if H is a normal subgroup of N and N is a normal subgroup of G it is not necessary that H be a normal subgroup of G . To make a transitive relation the concept of a subnormal group has been introduced and in the above situation H would be subnormal in G .

I am interested in investigating groups with many or all subgroups 2-subnormal. This is in some sense a step away from the situation in which every group is normal, yet the situation is much more complex. I am interested mainly in finite p -groups (groups of prime power order) with the property that all or most subgroups are 2-subnormal. These groups have been investigated for the last thirty years, but many questions remain unanswered. Some of this work involves construction of examples of groups using one of the computer algebra packages, MAGMA or GAP as well as more theoretical aimed at describing groups with the given properties. I would suggest as a project that a student use the literature to investigate groups with many subgroups 2-subnormal when the groups are 2-groups, 3-groups or 5-groups and find the similarities and differences when these primes are involved, and perhaps some explanation of the differences. Groups involving these primes can easily be constructed using one of the packages listed above.

Some of the literature is listed below.

C. Hobby, Finite groups with normal normalizer, *Canad. J. Math.* **20**, 1256-1260 (1968)

J. C. Lennox and S. E. Stonehewer, *Subnormal Subgroups of Groups*, Oxford Science Publications, 1987.

S. K. Mahdavianary, A classification of 2-generator p -groups, $p \geq 3$, with many groups 2-subnormal, *Arch. Math.* (Basel) **43**, 97-107 (1984).

E. A. Ormerod, Groups of Wielandt length two, *Math. Proc. Camb. Phil. Soc.* **110**, 229-244 (1991)

G. Parmeggiani, On finite p -groups of odd order with many subgroups 2-subnormal, *Communications in Algebra* **24**, 2707-2719 (1996)

2. Matrix-Product Codes over F_q .

One of the aims of mathematicians working in Coding Theory is to determine methods of constructing new codes from already existing codes. One of the well known ways of doing this is called the Plotkin $(u | u + v)$ -construction for binary codes. If C_1 is a code of length n , with K_1 codewords and minimum distance d_1 between codewords, and C_2 is a

code also of length, with K_2 codewords and minimum distance d_2 , Plotkin's construction yields a code of length $2n$, with K_1K_2 codewords and minimum distance $\min\{2d_1, d_2\}$. This construction gives a standard iterative method for constructing what is known as the Reed-Muller codes. These codes were instrumental in the transmission of pictures of Mars from Mariner 9. They are also used to correct errors in storage devices including compact discs, DVD and barcodes, in wireless and mobile communications, digital television and high-speed modems. There is a similar construction for ternary codes known as the $(u + v + w \mid 2u + v \mid u)$ – construction and this construction is iterated to produce a Main Sequence subfamily of the ternary Reed-Muller codes.

Given codes C_1, \dots, C_m of length n over \mathbf{F}_q , the finite field of q elements, and an $m \times n$ matrix A over \mathbf{F}_q it is possible to define a *matrix product code* $C = [C_1 \cdots C_m] \cdot A$ consisting of all matrix products $[c_1 \cdots c_m] \cdot A$. This generalises the $(u \mid u + v)$ –, $(u + v + w \mid 2u + v \mid u)$ – and other similar constructions. Matrix-product codes are studied using the tools of Linear Algebra, which provides a basis for a unified analysis of the size of the code C , the minimum Hamming distance of C , and the dual code C^\perp .

A project in this area would involve learning some elementary coding theory, understanding the construction of matrix-product codes over a finite field, determining the size of these codes and the minimum distances, constructing interesting examples of matrix-product codes and looking at applications of these codes.

Some literature on the subject is given below.

R. Hill, *A First Course in Coding Theory*, OUP, Oxford 1986.

Blackmore, T., Norton, G.H., *Matrix-Product Codes over \mathbf{F}_q* , AAECC 12 (2001) 6, 477-500.